

Determining the sign of Δ_{31} at long baseline neutrino experiments

Mohan Narayan and S. Uma Sankar

Department of Physics, I.I.T. , Powai, Mumbai 400076, India

(February 1, 2008)

Recently it is advocated that high intensity and low energy ($E_\nu \sim 2$ GeV) neutrino beams should be built to probe the (13) mixing angle ϕ to a level of a few parts in 10^4 . Experiments using such beams will have better signal to background ratio in searches for $\nu_\mu \rightarrow \nu_e$ oscillations. We propose that such experiments can also determine the sign of Δ_{31} even if the beam consists of *neutrinos* only. By measuring the $\nu_\mu \rightarrow \nu_e$ transitions in two different energy ranges, the effects due to propagation of neutrinos through earth's crust can be isolated and the sign of Δ_{31} can be determined. If the sensitivity of an experiment to ϕ is ε , then the same experiment is automatically sensitive to matter effects and the sign of Δ_{31} for values of $\phi \geq 2\varepsilon$.

Neutrino oscillations provide an elegant explanation for the observed deficits in solar and atmospheric neutrino fluxes. Both these deficits can be accounted for in a three flavor oscillation scheme. In this scheme, the oscillation probabilities, in general, depend on six parameters: two independent mass-squared differences Δ_{21} and Δ_{31} , three mixing angles ω , ϕ and ψ and a CP violating phase. Given the distance and energy scale of solar neutrinos, their oscillations depend only on one mass-squared difference $\Delta_{21} \sim 10^{-5}$ eV² and two mixing angles ω and ϕ [1,2]. Atmospheric neutrino oscillations are insensitive to a mass-squared difference as small as 10^{-5} eV². Hence Δ_{21} can be set to zero in the analysis of atmospheric neutrinos. In this approximation, the oscillation probabilities of atmospheric neutrinos depend on $|\Delta_{31}| \sim (1-7) \times 10^{-3}$ eV² [3] and two mixing angles ϕ and ψ [4]. The results of CHOOZ experiment [5] give the strong constraint $\sin^2 2\phi \leq 0.1$. This implies that either $\phi \leq 9^\circ$ or $\phi \geq 81^\circ$. The second possibility is ruled out by the constraints from both solar neutrino [6] and atmospheric neutrino analyses [7]. Thus ϕ is small and the atmospheric neutrino oscillations are almost $\nu_\mu \rightarrow \nu_\tau$ oscillations with a small $\nu_\mu \leftrightarrow \nu_e$ component. To explain the observed deficit of muon neutrinos, the mixing angle ψ should be close to $\pi/4$ [3].

Combining the data from solar, atmospheric neutrino experiments and CHOOZ we have the following pattern of neutrino masses and mixings: Two mass eigenstates are very close together and the third mass eigenstate is a little apart. This third state contains approximately equal admixture of ν_μ and ν_τ and a small admixture of ν_e . From a model building point of view, the crucial question is whether this third state is more massive than the two nearly degenerate states (Δ_{31} positive) or less massive (Δ_{31} negative).

Long baseline experiments are designed to observe ν_μ oscillations under controlled conditions. The baselines and energies of these experiments are chosen such that significant deficit of ν_μ flux will be observed if the neutrino parameters are in the range determined by Super-Kamiokande atmospheric neutrino analysis. Two different long baseline neutrino beams, one from Fermilab to Soudan [8] and another from CERN to Gran Sasso [9], are being constructed. In both cases the baseline length is 730 km and the beam consists of neutrinos only. The experiments downstream expect to observe neutrino oscillations via i) ν_μ disappearance ii) ν_τ appearance and iii) ν_e appearance. The high statistics of these experiments allow them to determine the energy distribution of ν_μ charged current (CC) events. By locating the minimum of ν_μ CC event distribution, one can determine $|\Delta_{31}|$ and $\sin^2 2\psi$ independently and to a precision of 10% [10]. But neither ν_μ disappearance nor ν_τ appearance data can give us information on the sign of Δ_{31} .

$\nu_\mu \rightarrow \nu_e$ oscillation probability in long baseline experiments is modified by the propagation of neutrinos through the matter of earth's crust. Matter effects [11] boost the oscillation probability for neutrinos if Δ_{31} is positive and suppress it if Δ_{31} is negative. The situation is reversed for anti-neutrinos. At neutrino factories, where ν_μ and $\bar{\nu}_\mu$ beams from muon storage rings will be available, one can measure $\nu_\mu \rightarrow \nu_e$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillation probabilities, isolate the difference induced by the matter effects and determine the sign of Δ_{31} [12]. However, the neutrino beam energies at these factories will be very high ($E_\nu \sim 20$ GeV) and hence one needs a very long baseline (of about 7000 km) so that the oscillation probability will not be too small. It may be a long time before this ambitious program is realized. Recently it was proposed that high intensity, low energy neutrino beams can be built with conventional techniques [13]. If these beams have peak flux around $E_\nu \sim 2$ GeV, then a 20 Kiloton detector at a distance of about 730 km will be capable of measuring values of ϕ as small as a few parts in 10^4 . In this letter we show that such a detector can determine the sign of Δ_{31} even if the beam consists of *neutrinos only*.

In three flavor scheme, $\nu_\mu \rightarrow \nu_e$ oscillation probability is given by

$$P_{\mu e} = \sin^2 \psi \sin^2 2\phi \sin^2 \left(\frac{1.27 \Delta_{31} L}{E} \right), \quad (1)$$

where Δ_{31} is in eV², the baseline length L is in km and the neutrino energy E is in GeV. From CHOOZ con-

straint, we see that this probability is atmost a few percent. In the long baseline experiments, the neutrinos propagate through earth's crust which has constant density of about 3 gm/cc. The oscillation probability with the inclusion of matter effects is given by [6,7]

$$P_{\mu e}^m = \sin^2 \psi \sin^2 2\phi^m \sin^2 \left(\frac{1.27 \Delta_{31}^m L}{E} \right). \quad (2)$$

Comparing Eq. (1) with Eq. (2), we note that the mixing angle ψ is unaffected by matter term and ϕ and Δ_{31} are replaced by their matter modified values ϕ^m and Δ_{31}^m respectively [4,6]. These are given by

$$\sin 2\phi^m = \frac{\Delta_{31} \sin 2\phi}{\Delta_{31}^m}, \quad \Delta_{31}^m = \sqrt{(\Delta_{31} \cos 2\phi - A)^2 + (\Delta_{31} \sin 2\phi)^2}. \quad (3)$$

Here A is the matter term and is given by

$$A = 2\sqrt{2}G_F N_e E = 0.76 \times 10^{-4} \rho \text{ (in gm/cc)} E \text{ (in GeV)}. \quad (4)$$

If Δ_{31} is positive, a resonance will occur for neutrinos at the energy

$$E_{res} = \frac{\Delta_{31} \cos 2\phi}{2.3 \times 10^{-4}}. \quad (5)$$

At this energy, $\Delta_{31} \cos 2\phi = A$ and $\sin 2\phi^m = 1$ [14]. Unfortunately, for a baseline of 730 km, $P_{\mu e}^m$ at E_{res} is not significantly greater than $P_{\mu e}$ at the same energy. This occurs because, at resonance, $\sin 2\phi^m$ is maximized but simultaneously Δ_{31}^m is minimized [2]. Hence nothing is gained by tuning the neutrino flux to peak around E_{res} . Away from the resonance, the energy dependence of $\sin^2 2\phi^m$ is different from that of $\sin^2(1.27\Delta_{31}^m L/E)$. So, in general, we can expect $P_{\mu e}^m$ to be different from $P_{\mu e}$. In Figure 1, we plotted $P_{\mu e}$ (middle line), $P_{\mu e}^m$ with Δ_{31} positive (upper line) and $P_{\mu e}^m$ with Δ_{31} negative (lower line). We chose $L = 730$ km, $\phi = 9^\circ$ and $|\Delta_{31}| = 3.5 \times 10^{-3} \text{ eV}^2$, which is Super-Kamiokande best-fit value. We see that for both signs of Δ_{31} , the maximum of $P_{\mu e}^m$ occurs close to the energy where $P_{\mu e}$ is maximum, that is where the phase of the oscillating term $1.27|\Delta_{31}|L/E = \pi/2$. We call this energy to be

$$E_{\pi/2} = \frac{2.54 |\Delta_{31}| L}{\pi}. \quad (6)$$

For $|\Delta_{31}| = 3.5 \times 10^{-3} \text{ eV}^2$, $E_{\pi/2} = 2 \text{ GeV}$, if $L = 730$ km.

Long baseline experiments will first measure $|\Delta_{31}|$ and $\sin^2 2\psi$ to about 10% precision. The next goal of neutrino experiments is to probe small values of ϕ by looking for $\nu_\mu \rightarrow \nu_e$ oscillations. In such a search, the neutrino beam energy should be tuned such that the oscillation signal

to background ratio is maximized. Neutrino crosssections increase linearly with neutrino energy and neutrino beam flux increases with the energy of the initial accelerated particle so neutrino event rates increase as E^p where $p > 1$. So, it seems as if high energy beams are more favorable to do neutrino physics than low energy beams. However, $\nu_\mu \rightarrow \nu_e$ oscillation probability falls as off as E^{-2} at energies much greater than $E_{\pi/2}$. Hence the increase in signal event rate, if any, is moderate. The background events come primarily from the neutral current events of neutrinos in the beam and the CC events of the 1% ν_e contamination of the beam. Both these are proportional to total event rate and increase as E^p . Hence the $\nu_\mu \rightarrow \nu_e$ signal to background ratio worsens as E^{-2} if one goes to energies $E \gg E_{\pi/2}$. In order to maximize this signal, it is advantageous to tune the energy such that flux of the neutrino beam peaks around $E_{\pi/2}$. A recent paper by B. Richter [13] also proposes such a tuning, where it is also advocated that construction of high intensity and low energy neutrino beams can be the next step in the effort to measure small neutrino parameters. At such beams the signal to background ratio for $\nu_\mu \rightarrow \nu_e$ oscillations, will be much larger than in the case of high energy beams. Such beams can be used to probe values of ϕ as small as a few parts in 10^4 . They can also be used to determine the sign of Δ_{31} , even if they consist of neutrinos only.

We note from Figure 1 that $P_{\mu e}^m$ peaks close to where $P_{\mu e}$ peaks, that is close to $E = E_{\pi/2}$. In addition, we can make three important observations:

1. Above 4 GeV (or above $2E_{\pi/2}$), matter effects have negligible effect on $\nu_\mu \rightarrow \nu_e$ oscillation probability. For both signs of Δ_{31} , $P_{\mu e}^m$ is very close to $P_{\mu e}$ for energies above $2E_{\pi/2}$. Hence by selecting events with $E > 2E_{\pi/2}$, one can directly measure ϕ without worrying about matter effects.
2. Matter effects are most dominant in the neighbourhood of $E_{\pi/2}$. In this neighbourhood, $P_{\mu e}^m$ is about 25% greater than $P_{\mu e}$ if Δ_{31} is positive and is smaller by about the same factor if Δ_{31} is negative. This is true for all values of $|\Delta_{31}|$ and ϕ [7].
3. For fixed value $|\Delta_{31}|/E$, matter effects cause a larger change for larger values of L . This occurs for the following reason. In the neighbourhood of $E_{\pi/2}$, the phase of the oscillating term is $\pi/2$. However, $E_{\pi/2}$ is larger if L is larger and $\sin^2 2\phi_m$ is larger for larger energies. So matter term causes a larger change in $P_{\mu e}^m$ around $E_{\pi/2}$ for larger values of L .

To observe the differences induced by matter effects, one needs to measure $\nu_\mu \rightarrow \nu_e$ oscillation probability in the lower energy range $0 < E_\nu < 2E_{\pi/2}$. Using the value of ϕ measured from the data of the higher energy range $E_\nu > 2E_{\pi/2}$, one can predict the expected number of

events for the lower energy range, for Δ_{31} positive and for Δ_{31} negative. By comparing the results of the lower energy range with the predictions, one can determine the sign of Δ_{31} . Since the vacuum value of ϕ is unknown, one needs a minimum of two measurements to determine the sign of Δ_{31} . Usually these are taken to be $\nu_\mu \rightarrow \nu_e$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillation probabilities. But, as we argued above, $\nu_\mu \rightarrow \nu_e$ event samples in the lower and higher energy ranges also can be used to distinguish between the two possible signs of Δ_{31} . The neutrino beam should have reasonable width in energy (say $0 < E_\nu < 4E_{\pi/2}$) so that there will be substantial number of events both in the lower and higher energy regions.

The following procedure may be used to determine the sign of Δ_{31} . From the experiment we will get two pieces of data, N_{eh} from the higher energy region and N_{el} from the lower energy region. We wish to check whether the hypothesis of positive or negative Δ_{31} fits the data better. Let $N_{eh}^{m+}(\phi)$ and $N_{eh}^{m-}(\phi)$ are the theoretical expectation for the number of electron events in the higher energy region for positive and negative Δ_{31} respectively. These two numbers will be functions of the mixing angle ϕ which, as yet, is unknown. Similarly, $N_{el}^{m+}(\phi)$ and $N_{el}^{m-}(\phi)$ are the theoretical expectations for the lower energy region. Now we define two different χ^2 s,

$$\chi_+^2 = \frac{(N_{eh} - N_{eh}^{m+})^2}{N_{eh}} + \frac{(N_{el} - N_{el}^{m+})^2}{N_{el}},$$

$$\chi_-^2 = \frac{(N_{eh} - N_{eh}^{m-})^2}{N_{eh}} + \frac{(N_{el} - N_{el}^{m-})^2}{N_{el}}. \quad (7)$$

Both these χ^2 s are functions of ϕ and by varying ϕ they can be minimized. If Δ_{31} is positive, we will have $\chi_+^2|_{min} \ll \chi_-^2|_{min}$ and vice-versa if Δ_{31} is negative. This strong inequality between the respective minima of the two χ^2 s must occur if our present understanding of effect of matter term on the propagation of neutrinos is correct.

Let us briefly consider the ability of long baseline neutrino experiments to determine the sign of Δ_{31} . Suppose the minimum value of ϕ an experiment can measure is ε . In the following, we assume that the neutrino beam has the same spectrum as that of the low energy option of MINOS [10]. We will also take $|\Delta_{31}| = 3.5 \times 10^{-3}$ eV² and the baseline length to be $L = 730$ km, so that $E_{\pi/2} = 2$ GeV. In such a case, the $\nu_\mu \rightarrow \nu_e$ oscillation signal events are split 3 : 1 between the lower energy range (0 – 4 GeV) and the high energy range (> 4 GeV). The ν_μ flux, and hence the background events, are split in the ratio 4 : 6 for the same two ranges. The signal to background ratio in the lower energy range is about 5 times better than in the higher energy range. This illustrates our earlier point that neutrino beams with energy tuned to $E_{\pi/2}$ are better than high energy beams for observing $\nu_\mu \rightarrow \nu_e$ oscillations. Let us assume that, to be sensitive to $\phi = \varepsilon$, the experiment must be capable of observing N

ν_e -CC events above the background. Of these, $3N/4$ will be in the lower energy range and $N/4$ will be in the higher energy range. If the value of ϕ is 2ε , then this experiment will see $4N$ ν_e -CC events in case of vacuum $\nu_\mu \rightarrow \nu_e$ oscillations. Of these, $3N$ will be in the lower energy range and N will be in the higher energy range. The number of events in the higher energy range is not affected by matter effects and this number, N , is large enough to be detectable above the background. Matter effects boost the events in the lower energy range to $3.75N$ if Δ_{31} is positive and suppress them to $2.25N$ if Δ_{31} is negative. In each case, the difference induced by the matter effects is larger than the background in this energy range, which is less than half the total background. Hence, any long baseline neutrino experiment which sensitive to $\phi \simeq \varepsilon$ is automatically sensitive to the sign of Δ_{31} if $\phi \simeq 2\varepsilon$.

In conclusion, we showed that neutrino beams with energy tuned to $E_{\pi/2}$ have better signal to background ratios than the high energy neutrino beams. We also showed that a future high intensity and low energy neutrino experiment, which can measure values of (13) mixing angle ϕ as small as a few parts in 10^4 , can automatically measure the sign of Δ_{31} for values of ϕ greater than double the sensitivity of the experiment even if the beam consists of neutrinos only.

We would like to thank Prof. A. Bettini and K.R.S. Balaji for discussions. We would also like to thank the Department of Science and Technology, Govt. of India for supporting this research.

-
- [1] T. K. Kuo and J. Pantaleone, Phys. Rev. **D35**, 3432 (1987).
 - [2] A. S. Joshipura and M. V. N. Murthy, Phys. Rev. **D37**, 1374 (1988).
 - [3] H. Sobel, Talk presented at *Neutrino 2000*, Sudbury, Canada, June 2000.
 - [4] J. Pantaleone, Phys. Rev. **D49**, 2152 (1994).
 - [5] CHOOZ Collaboration: M. Appollonio *et al*, Phys. Lett. **466B**, 415 (1999).
 - [6] Mohan Narayan and S. Uma Sankar, Phys. Rev. **D61**, 013003 (2000).
 - [7] P. Lipari, Phys. Rev. **D61**, 113004 (2000).
 - [8] MINOS technical Design Report, Nu-MI-L-337, October 1998.
 - [9] A. Rubbia, Talk presented at *Neutrino 2000*, Sudbury, Canada, June 2000.
 - [10] David A. Petyt, *Physics Potential of the three Ph2 beam designs*, MINOS preprint Nu-MI-L 612 (April, 2000).
 - [11] L. Wolfenstein, Phys. Rev. **D17**, 2369 (1978); *ibid* **D20**, 2634 (1979).
 - [12] C. Albright *et al*, *Physics at Neutrino Factory*, hep-ex/0008064
 - [13] B. Richter, SLAC-PUB-8587, hep-ph/0008222

- [14] S. P. Mikheyev and A. Yu. Smirnov, *Yad. Fiz.* **42**, 1441 (1985) [*Sov. J. Nucl. Phys.* **42**, 913 (1985)]; *Nuovo Cimento* **C9**, 17 (1986).

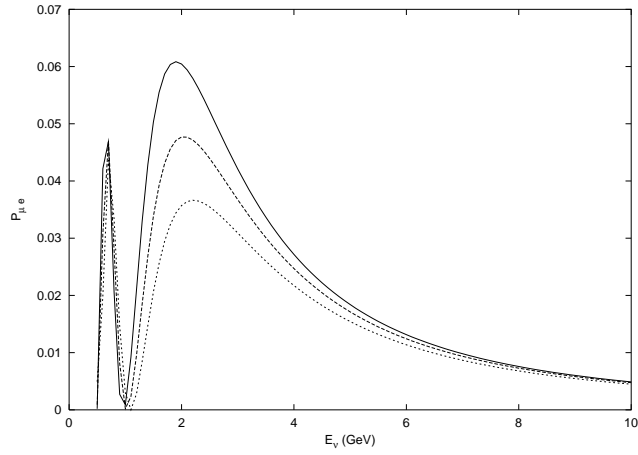


FIG. 1. $\nu_\mu \rightarrow \nu_e$ oscillation probabilities vs E for $|\Delta_{31}| = 3.5 \times 10^{-3} \text{ eV}^2$, $\sin^2 2\phi = 0.1$ and $L = 730 \text{ km}$. The middle line is $P_{\mu e}$, the upper line is $P_{\mu e}^m$ with Δ_{31} positive and the lower line is $P_{\mu e}^m$ with Δ_{31} negative.